Design of Cyber-Physical Systems Using Passivity/Dissipativity

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Acknowledgements

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Outline

- **A Brief Introduction to Cyber-Physical Systems (CPS).**
  What are they and why are they important and interesting?

-***Studying CPS using Passivity and Dissipativity.***

-***Background on Passivity and Passivity Indices.***
Preserving passivity when interconnecting systems.

-***Passivation using Passivity Indices. Performance.***
-***Application to Automotive and Human-Operator Applications.***
-***System Approximations and Passivity/Dissipativity.***

-***Switched/Hybrid Systems, Networked, Discrete Event Systems.***

-***Systems with Symmetries and approximate Symmetries.***

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Cyber-Physical Systems (CPS)

-As computers become ever-faster and communication bandwidth ever-cheaper, computing and communication capabilities will be embedded in all types of objects and structures in the physical environment.

-***Cyber-physical systems (CPS)*** are physical, biological and engineered systems whose ***operations are monitored, coordinated,* controlled *and integrated by a computing and communication core***.

-This intimate coupling between the cyber and physical will be manifested from the nano-world to large-scale wide-area systems of systems. And at multiple time-scales.

-Applications with enormous societal impact and economic benefit will be created. Cyber-physical systems will transform how we interact with the physical world just like the Internet transformed how we interact with one another.

-**We should care about CPS because our lives depend on them**

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Technological and Economic Drivers

•The decreasing cost of computation, networking, and sensing.

• A variety of social and economic forces will require us to use national infrastructures more efficiently.

• Environmental pressures will mandate the rapid introduction of technologies to improve energy efficiency and reduce pollution.

•As the national population ages, we will need to make more efficient use of our health care systems, ranging from facilities to medical data and information.

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PCAST Report

Leadership Under Challenge:
Information Technology R&D in a Competitive World
An Assessment of the Federal Networking and Information Technology R&D Program
President's Council of Advisors on Science and Technology
August 2007

New Directions in Networking and Information Technology (NIT)

Recommendation: No 1 Funding Priority:
NIT Systems Connected with the Physical World

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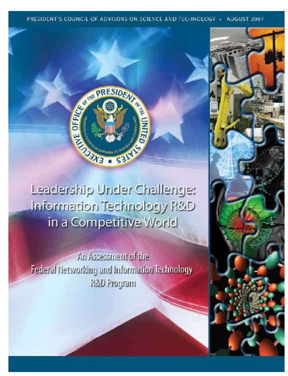
PCAST Report

Chapter 4 in Report –Technical Priorities for NIT R&D

1. NIT Systems Connected with the Physical World
2. Software
3. Data, Data Stores, and Data Streams
4. Networking
5. High End Computing
6. Cyber Security and Information Assurance
7. Human-Computer Interaction
8. NIT and the Social Sciences



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CPS Characteristics

What cyber physical systems have as defining characteristics:

- Cyber capability (i.e. networking and computational capability) in every physical component
- They are networked at multiple and extreme scales
- They are complex at multiple temporal and spatial scales.
- They are dynamically reorganizing and reconfiguring
- Control loops are closed at each spatial and temporal scale. Maybe human in the loop.
- Operation needs to be dependable and certifiable in certain cases
- Computation/information processing and physical processes are so tightly integrated that it is not possible to identify whether behavioral attributes are the result of computations (computer programs), physical laws, or both working together.

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Passivity, Symmetry and CPS

-Heterogeneity causes major challenges. Also dynamic changes (verification, security implications). In addition network uncertainties, time-varying delays, data rate limitations, packet losses.

-**How do we guarantee desirable properties in a network of heterogeneous systems which may change dynamically, and expand or contract? Passivity inequalities. Comparison with Lyapunov stability.**

-**Can we start with a system and grow it in particular ways to preserve its properties? Symmetry.**

-**We impose passivity constraints on the components (also symmetry), and the design can accommodate heterogeneity and network effects. Approach also useful in human-interaction.**

- **NSF CPS Large Project: "Science of Integration of CPS" (with Vanderbilt, Maryland, GM R&D).**

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Background on Passivity

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Definition of Passivity in Continuous-time

- Consider a continuous-time nonlinear dynamical system

$$\dot{x} = f(x,u)$$
$$y = h(x,u).$$

- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0 (for all x)* such that

$$\int_{t_1}^{t_2} u^T(t)y(t)dt + V(x(t_1)) \geq V(x(t_2))$$

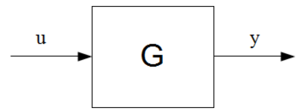
for all $t_2 \geq t_1$ and input $u(t) \in U$.

- When V(x) is continuously differentiable, it can be written as:

$$u^T(t)y(t) \geq \dot{V}(x(t))$$

Alternative Definition of Passivity

- Passivity is inherently an input-output property.



- It is independent of internal representation. An alternative definition is that for all inputs *u(t)* ϵ *U* and times *T*, there exists a *β* to satisfy the following inequality

$$\int_0^T u^T(t)y(t)dt \geq -\beta.$$

- When the system has zero initial conditions the inequality reduces to

$$\int_0^T u^T(t)y(t)dt \geq 0.$$

Examples of Passive Systems

- G1 is a passive system if the pole is negative (*a*≥*0*) but not passive for *a<0*

$$G_1(s) = \frac{1}{s+a}$$

- G2 is not passive for any *a* because of the negative gain

$$G_2(s) = \frac{-1}{s+a}$$

- Even a stable, minimum phase system can be non-passive if the phase shift is too large (G3)

$$G_3(s) = \frac{s+5}{s^2+2s+2}$$

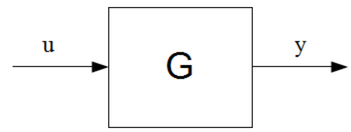
- G3 would have been passive if the zero were closer to the origin (G4)

$$G_4(s) = \frac{s+0.5}{s^2+2s+2}$$

Passivity in Discrete-time

- Passivity can also be defined for discrete-time systems. Consider a nonlinear discrete time system

$$x(k+1) = f(x(k), u(k))$$
$$y(k) = h(x(k), u(k)).$$



- This system is *passive* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\sum_{k=k_1}^{k_2} u^T(k)y(k) + V(x(k_1)) \geq V(x(k_2))$$

for all k_1, k_2 and all inputs $u(k) \in U$.

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Extended Definitions of Passivity

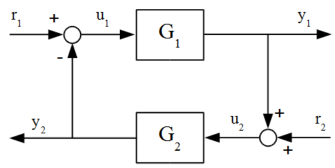
Passive	$u^T y \geq \dot{V}(x)$
Lossless	$u^T y = \dot{V}(x)$
Strictly Passive	$u^T y \geq \dot{V}(x) + \psi(x)$
Strictly Output Passive	$u^T y \geq \dot{V}(x) + \varepsilon y^T y$
Strictly Input Passive	$u^T y \geq \dot{V}(x) + \delta u^T u$

- Note that *V(x)* and *Ψ(x)* are positive definite and continuously differentiable. The constants ε and δ are positive. These equations hold for all times, inputs, and states.

Interconnections of Passive Systems

- One of the strengths of passivity is when systems are interconnected. Passive systems are stable and passivity is preserved in many practical interconnections.

- For example, the negative feedback interconnection of two passive systems is passive.



- If $u_1 \to y_1$ and $u_2 \to y_2$ are passive then the mapping $r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \to y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ is passive

- Note: the other internal mappings ($u_1 \to y_2$ and $u_2 \to y_1$) will be stable but may not be passive

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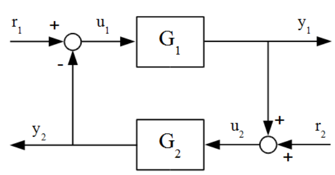
Interconnections of Stable Systems

- Compared with passive systems, the feedback interconnection of two stable systems is not always stable



- One notable special case is the small gain theorem where if G_1 and G_2 are finite-gain L_2 stable with gains γ_1 and γ_2 then the interconnection is stable if $\gamma_1 \gamma_2 < 1$.

- Both Passivity theory and the small gain theorem are special cases of larger frameworks including the conic systems theory and the passivity index theory.

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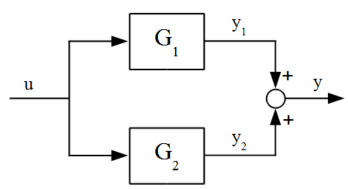
Stability of Passive Systems

- Strictly passive systems (*ψ(x)>0*) are asymptotically stable
- Output strictly passive systems (*δ>0*) are L_2 stable
- The following results hold in feedback
  - Two passive systems → passive and stable loop
  - Passive system and a strictly passive system → asymptotically stable loop
  - Two output strictly passive systems → L_2 stable loop
  - Two input strictly passive systems (*ε>0*) → L_2 stable loop

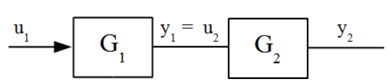
$$u^T y \geq \dot{V}(x) + \varepsilon u^T u$$

Other Interconnections

- The parallel interconnection of two passive systems is still passive



- However, this isn't true for the series connection of two systems



- For example, the series connection of any two systems that have 90°of phase shift have a combined phase shift of 180°

Dissipativity, conic systems, and passivity indices

[McCourt and Antsaklis, ISIS-2009-009]
[Kottenstette, McCourt, Xia and Antsaklis, 2014 Automatica]

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Definition of Dissipativity (CT)

- This concept generalizes passivity to allow for an arbitrary energy supply rate *ω(u,y).*
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function $V(x) \geq 0$ such that

$$\int_{t_1}^{t_2} \omega(u,y)dt \geq V(x(t_2)) - V(x(t_1))$$

for all t_1, t_2 and the input $u(t) \in U$.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Qy + 2y^T Su + u^T Ru.$$

- *QSR dissipative systems are L_2 stable when Q<0*

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Definition of Dissipativity (DT)

- The concept of dissipativity applies to discrete time systems for an arbitrary supply rate *ω(u,y).*
- A system is *dissipative* with respect to supply rate *ω(u,y)* if there exists a continuous storage function *V(x) ≥ 0* such that

$$\sum_{k=k_1}^{k_2} \omega(u,y) \geq V(x(k_2)) - V(x(k_1))$$

for all k_1, k_2 and the input $u(k) \in U$.

- A special case of dissipativity is the QSR definition where the energy supply rate takes the following form:

$$\omega(u,y) = y^T Qy + 2y^T Su + u^T Ru.$$

- Dissipative DT systems are stable when $Q<0$

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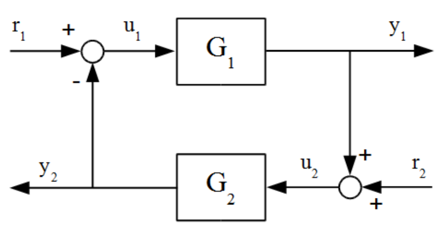
Stability using dissipativity

- Dissipative systems may not be passive or stable
- Stability of the feedback interconnection of two dissipative systems can be assessed



- If G_1 is dissipative with (Q_1,S_1,R_1) and G_2 is dissipative with (Q_2,S_2,R_2), the feedback interconnection of the two systems is stable if the following LMI is satisfied

$$\begin{bmatrix} Q_1 + R_2 & S_1 - S_2^T \\ S_1^T - S_2 & Q_2 + R_1 \end{bmatrix} \leq 0$$

- This can also be seen as a control design tool. Say (Q_1,S_1,R_1) are known then stabilizing (Q_2,S_2,R_2) can be found. A controller can then be designed from (Q_2,S_2,R_2) to stabilize

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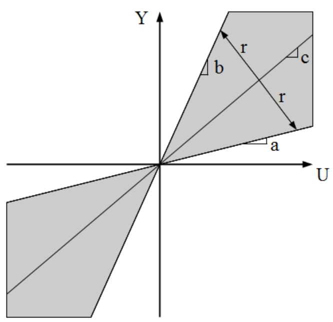
Conic Systems

- A conic system is one whose input-output behavior is constrained to lie in a cone of the *U x Y* inner product space



- A system is conic if the following dissipative inequality holds for all $t_2 \geq t_1$

$$\int_{t_1}^{t_2} \left[(1+\frac{a}{b}) u^T y - a u^T u - \frac{1}{b} y^T y \right] dt \geq V(x(t_2)) - V(x(t_1))$$

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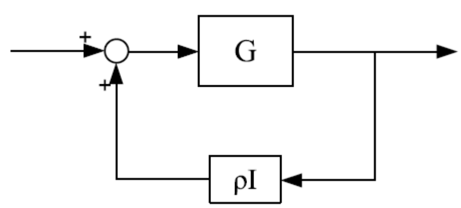
Output Feedback Passivity Index

The output feedback passivity index (OFP) is the largest gain that can be put in positive feedback with a system such that the interconnected system is passive.



Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y dt$$

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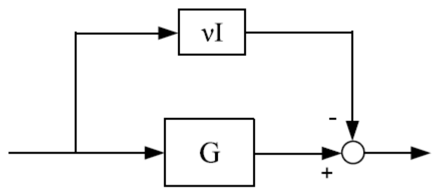
Input Feed-Forward Passivity Index

The input feed-forward passivity index (IFP) is the largest gain that can be put in a negative parallel interconnection with a system such that the interconnected system is passive.



Equivalent to the following dissipative inequality holding for G

$$\int_{t_1}^{t_2} u^T y dt \geq V(x(t_2)) - V(x(t_1)) + \nu \int_{t_1}^{t_2} u^T u dt$$

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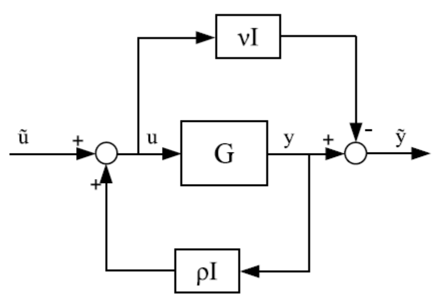
Simultaneous Indices

When applying both indices the physical interpretation as in the block diagram



Equivalent to the following dissipative inequality holding for G

$$(1+\rho\nu) \int_{t_1}^{t_2} u^T y dt \geq V(x(t_2)) - V(x(t_1)) + \rho \int_{t_1}^{t_2} y^T y dt + \nu \int_{t_1}^{t_2} u^T u dt$$

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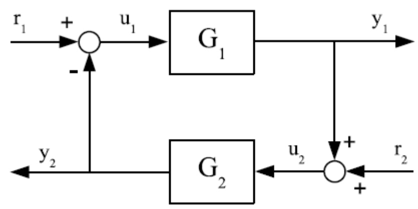
Stability

We can assess the stability of an interconnection using the indices for G_1 and G_2



G_1 has indices ρ_1 and ν_1

G_2 has indices ρ_2 and ν_2

The interconnection is L_2 stable if the following matrix is positive definite

$$\begin{bmatrix} (\rho_1+\nu_2)I & \frac{1}{2}(\rho_2\nu_2-\rho_1\nu_1)I \\ \frac{1}{2}(\rho_2\nu_2-\rho_1\nu_1)I & (\rho_2+\nu_1)I \end{bmatrix} > 0$$

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Networked Passive Systems

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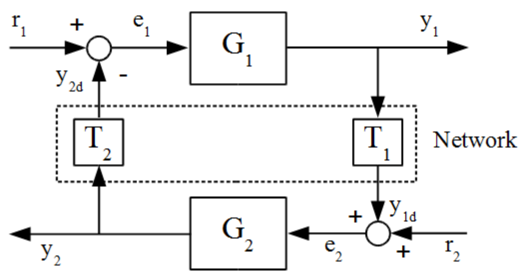
Networked Systems

- Motivating Problem: The feedback interconnection of two passive systems is passive and stable. However, when the two are interconnected over a delayed network the result is not passive so stability is no longer guaranteed. How do we recover stability?



The systems G_1 and G_2 are interconnected over a network with time delays T_1 and T_2

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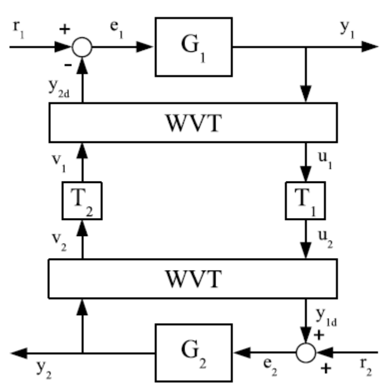
Stability of Networked Passive Systems

- One solution to interconnecting passive systems over a delayed network is to add an interface between the systems and the network
- The wave variable transformation forces the interconnection to guarantee stability for arbitrarily large time delays
- The WVT is defined below



$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_{2d} \end{bmatrix} \qquad \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \frac{1}{\sqrt{2b}} \begin{bmatrix} bI & I \\ bI & -I \end{bmatrix} \begin{bmatrix} y_{1d} \\ y_2 \end{bmatrix}$$

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Passivity and Dissipativity for Switched Systems

[McCourt and Antsaklis, 2012 ACC]
[McCourt and Antsaklis, 2010 CDC]
[McCourt and Antsaklis, 2010 ACC]

Passivity for Switched Systems

- The notion of passivity has been defined for switched systems

$$\dot{x} = f_\sigma(x,u)$$
$$y = h_\sigma(x,u)$$

A *switched system* is *passive* if it meets the following conditions

1. Each subsystem i is passive when active:

$$\int_{t_1}^{t_2} u^T y dt \geq V_i(x(t_2)) - V_i(x(t_1))$$

2. Each subsystem i is dissipative w.r.t. ω_j^i when inactive:

$$\int_{t_1}^{t_2} \omega_j^i(u,y,x,t)dt \geq V_j(x(t_2)) - V_j(x(t_1))$$

3. There exists an input u so that the cross supply rates (ω_j^i) are integrable on the infinite time interval.

[McCourt & Antsaklis 2010 ACC, 2010 CDC]

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Dissipativity for Switched Systems

Definition of Dissipativity in Discrete-time

A discrete-time switched system is dissipative if for each subsystem i there exists a positive function V_i such that the following conditions hold

1. Each subsystem i is dissipative while it is active with respect to $\omega_i(u, y)$:

$$\omega_i\big(u(t), y(t)\big) \geq V_i(x(t+1)) - V_i(x(t))$$

2. Each subsystem is dissipative when inactive with respect to $\omega_{ij}\,(u, y, x, t)$ for each active subsystem j:

$$\omega_{ij}\,(u(t), y(t), x(t), t(t)) \geq V_i(x(t+1)) - V_i(x(t))$$

Stability for Dissipative Discrete-time Systems

Theorem. *Dissipative switched systems are stable when* $\omega_i < 0$ *for all* i *and there exists an infinitely summable function* $\phi(t)$ *so that the inactive energy is bounded,*

$$\phi(t) \geq \omega_{ij}\,(u, y, x, t).$$

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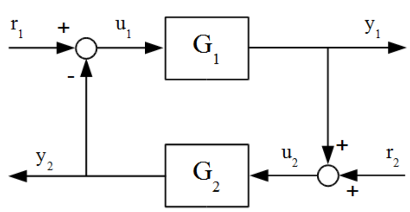
QSR Dissipativity for Switched Systems

- QSR dissipativity uses a quadratic supply rate to capture energy

$$\omega_i(u, y) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{vmatrix} Q_i & S_i \\ S_i^{\,T} & R_i \end{vmatrix} \begin{bmatrix} y \\ u \end{bmatrix}$$

- Stability of switched systems can be assessed using Q_i



- Dissipativity of the feedback interconnection of two switched systems can be assessed with Q_i, S_i, R_i of both systems
- Large scale systems can be analyzed or designed using QSR dissipativity to ensure that the entire system is stable
- When dealing with passive switched systems ($Q_i = 0$, $S_i = {}^1\!/_2\, I$, $R_i = 0$), any sequential combination of systems in feedback or parallel can be shown to be passive and stable

[McCourt &Antsaklis 2012 ACC]

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Dissipativity for Switched Systems

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A discrete-time switched system is dissipative if for each subsystem i there exists a positive function V_i such that the following conditions hold

1. Each subsystem i is dissipative while it is active with respect to $\omega_i(u, y)$:

$$\omega_i\big(u(t), y(t)\big) \geq V_i(x(t+1)) - V_i(x(t))$$

2. Each subsystem is dissipative when inactive with respect to $\omega_{ij}\ (u, y, x, t)$ for each active subsystem j:

$$\omega_{ij}\ (u(t), y(t), x(t), t(t)) \geq V_i(x(t+1)) - V_i(x(t))$$

Stability for Dissipative Discrete-time Systems

Theorem. *Dissipative switched systems are stable when* $\omega_i < 0$ *for all* i *and there exists an infinitely summable function* $\phi(t)$ *so that the inactive energy is bounded,*

$$\phi(t) \geq \omega_{ij}\ (u, y, x, t).$$

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Passivity and Dissipativity in Networked Switched Systems

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Computational and Experimental Methods for Passivity and Dissipativity Determination

[McCourt and Antsaklis, 2014 ACC]
[McCourt and Antsaklis, ISIS-2013-008]
[Wu, McCourt and Antsaklis, ISIS-2013-002]

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Showing Passivity and Dissipativity

- Passivity and dissipativity are powerful properties for analysis and synthesis of dynamical systems.
- Requires finding a positive storage function V and an appropriate ω in the case of dissipativity.

$$\int_{t_1}^{t_2} \omega(u(t), y(t))dt + V\,(x(t_1)) \geq V(x(t_2))$$

- In a switched system with m subsystems, dissipativity requires finding m storage functions and $\sim m^2$ dissipative rates
- In the worst case, this is a search similar to finding a Lyapunov function
- In many practical cases, this can be automated so that a program can generate an energy storage function (for LTI systems this is done using LMIs)

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LMI Methods – Passivity

- There are computational methods to find storage functions. For LTI passive systems, can always assume there exists a quadratic storage function

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

$$\begin{bmatrix} A^T P + PA & PB - C^T \\ B^T P - C & -D - D^T \end{bmatrix} \leq 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P & A^T PB - C^T \\ B^T PA - C & B^T PB - D - D^T \end{bmatrix} \leq 0$$

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LMI Methods – QSR Dissipativity

- The same can be done to demonstrate that an LTI system is QSR dissipative. Once again, a quadratic storage function is used

$$V(x) = \frac{1}{2} x^T P x \quad \text{where} \quad P = P^T > 0.$$

- For continuous-time system this leads to the following LMI

$$\begin{bmatrix} A^T P + PA - C^T QC & PB - C^T QD - C^T S \\ B^T P - D^T QC - S^T C & -D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

- In discrete-time the LMI becomes the following

$$\begin{bmatrix} A^T PA - P - C^T QC & A^T PB - C^T QD - C^T S \\ B^T PA - D^T QC - S^T C & B^T PB - D^T QD - S^T D - D^T S - R \end{bmatrix} \leq 0$$

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LMI Example

- Consider the following linear system

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ -8 & -1.5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} x + 0.5u$$

- The LMI solver in MATLAB determines the system to be passive with storage function

$$V(x) = \frac{1}{2} x^T P x \quad \text{with} \quad P = \begin{bmatrix} 12.9 & 1 \\ 1 & 1 \end{bmatrix}$$

- It can also be shown to be QSR dissipative (with respect to *Q= -.5*, *S=.5*, *R=0*) with storage function

$$V(x) = \frac{1}{2} x^T P x \quad \text{with} \quad P = \begin{bmatrix} 4.75 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

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SOS Methods

- For nonlinear systems, there isn't a general method to find storage functions
- For polynomial nonlinear systems, there are sum of squares (SOS) methods

- Any problem that can be expressed as a search for a positive polynomial function with polynomial constraints can be solved using SOS optimization
- Consider a system

$$\begin{aligned} x(t+1) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned}$$

where *f*, *g*, and *h* are polynomials.

- Need to search for a polynomial storage function V(x) to show dissipativity for a given Q, S, and R

[McCourt and Antsaklis, 2014 ACC]

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SOS Methods – Dissipativity

- The form of the storage function is based on the order of the desired storage function and a vector of lower order monomials
$$z = [x_1\ x_2\ x_1^2\ x_1x_2\ x_2^2\ \ x_1^3\ \ldots]^T$$
- The search is for the coefficients of the monomials to make up the polynomial storage function $$V(x) = z^T Lz$$
- This can be formulated as a sum of squares optimization problem to find the elements of $L=\{\ell_{ij}\}$
- The optimization problem is to minimize a user chosen linear combination of the entries of L subject to $V(x) = z^T Lz$ being polynomial and the dissipative inequality holding for the given system
- This opens up a class of nonlinear systems that can be identified as passive or dissipative in an automated way

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SOS Example

- Consider the following unstable (non-passive) nonlinear system
$$\dot{x} = \begin{bmatrix} -1.2x_1 + .65x_2 \\ .2x_2 - x_1^3 + u \end{bmatrix}$$
$$y = x_2$$
- This system isn't passive but it is QSR dissipative (with respect to *Q= 0.4, S=0.5, R=-0.9*) with storage function
$$V(x) = 0.49x_1^2 + 0.91x_2^2 + 0.69x_2^4$$
- This system can be feedback stabilized with another system with (for example) *Q= 0, S=0.5, R=1.* A simple example would be
$$\dot{x} = u$$
$$y = x + u$$

(continued)

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Experimental Methods

- The passivity levels (indices) may also be determined experimentally if input/output simulations or physical experimental settings are available.

-Restricted to classes of inputs.

-If certain system parameters are adjustable the passivity indices may be optimized using non-derivative based optimization;

-Performance can also be adjusted.

[Wu, McCourt and Antsaklis, ISIS-2013-002]

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Feedback Passivation Using Passivity Levels

[Zhu, Xia and Antsaklis, 2014 ACC]

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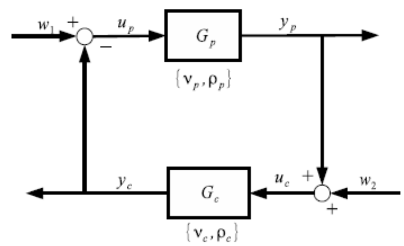
Problem Formulation

- Assume that the passivity levels for subsystems are known,



- Two problems are considered:
  - Passivity analysis – to determine the passivity levels of the interconnected system;
  - Passivation synthesis – to render a non-passive system passive (called passivation) through feedback.

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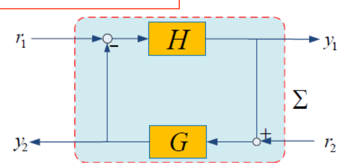
Existing Results

- Feedback Interconnections of IF-OFP systems:

1) If system H and G are passive, then system Σ is passive.
2) If system H and G are output strictly passive (OSP), then system Σ is OSP;
3) If system H is IF-OFP(ϵ_1, δ_1) and system G is IF-OFP(ϵ_2, δ_2), where $\epsilon_1 + \delta_2 > 0$, $\epsilon_2 + \delta_1 > 0$, then system Σ is finite gain stable (FGS). □



- Restrictions:
  - either assume passivity of subsystems ;
  - or focus on stability of the interconnected system.

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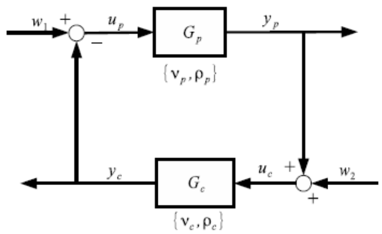
Main Results (1)

- Assume that the passivity levels for subsystems are known,



- Theorem: the passivity levels for the interconnected system satisfy:

$$\boxed{\begin{cases} \epsilon < \min\{\nu_p, \nu_c\} \\ \delta \le \min\left\{\rho_c - \frac{\epsilon\nu_p}{\nu_p - \epsilon}, \rho_p - \frac{\epsilon\nu_c}{\nu_c - \epsilon}\right\} \end{cases}}$$

- Comment: in general, the precise values of the passivity levels are difficult to obtain (especially for nonlinear systems).

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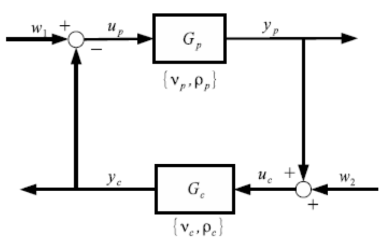
Main Results (2)

- Assume that the passivity levels for subsystems are known,



$$\boxed{\begin{aligned} w_2 &= 0 \\ \nu_p &\ge 0 \\ \rho_c &\ge 0 \\ \nu_p + \rho_c &> 0. \end{aligned}}$$

- Theorem: the passivity levels for the interconnected system satisfy:

$$\boxed{\begin{cases} \epsilon \le \frac{\nu_p\rho_c}{\nu_p + \rho_c} \\ \delta \le \nu_c + \rho_p \end{cases}}$$

- Note that when $w_2 = 0$, then the passivity of the system that maps w_1 to y_p can be guaranteed even if one subsystem is non-passive.

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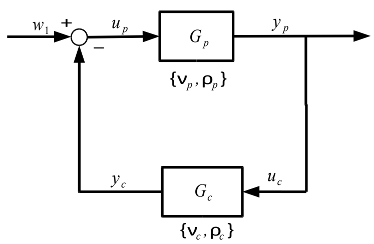
Passivation

- If one of the system is non-passive, say e.g. $\rho_p < 0$, how to passivate the system (from w_1 to y_p using passivity levels?



- The controller need to satisfy the following conditions:

$$\boxed{\rho_c \geq 0 \quad \nu_c > -\rho_p}$$

- The closed-loop system has passivity levels:

$$\boxed{\epsilon \leq \frac{\nu_p \rho_c}{\nu_p + \rho_c} \qquad \delta \leq \nu_c + \rho_p}$$

- Comment: can design controller based on desired ϵ and δ.

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Example (1) – Linear Systems

- The plant G_p is linear system with $\rho_p = -0.2$ and $v_p = 0$.

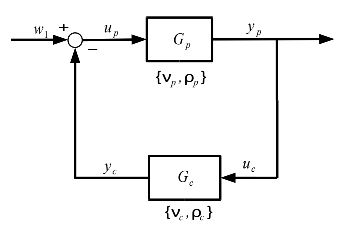
$$G_p = \frac{s+0.5}{s-0.1}$$

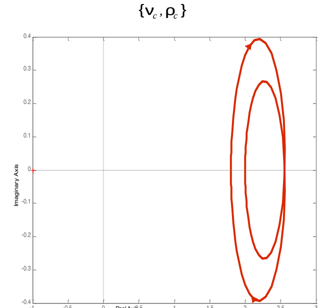
- The controller G_c is a linear controller with $\rho_c = 0$ and $v_c = 1$.

$$G_c = \frac{s+4}{s+2}$$

- The closed loop system has the passivity levels $\rho = 0.8$ and $\nu = 0$.
- The closed-loop transfer function is OSP with passivity index 1.8.

$$\frac{s^2+2.5s+1}{2s^2+6.4s+1.8}$$





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Example (2) – Nonlinear Systems

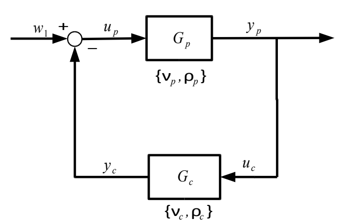
- The plant G_p is a nonlinear system with $\rho_p = -0.5$ and $\nu_p = 1.5$.

$$G_p \begin{cases} \dot{x}_1 & = & x_2 \\ \dot{x}_2 & = & -0.5x_1^3 + 0.5x_2 + 2u_p \\ y_p & = & x_2 + u_p \end{cases}$$

- The controller G_c is a linear controller with $\rho_c = 0$ and $\nu_c = 1$.

$$G_c = \frac{s+4}{s+2}$$

- The closed-loop system has the passivity levels $\rho = 0.5$ and $\nu = 0$.



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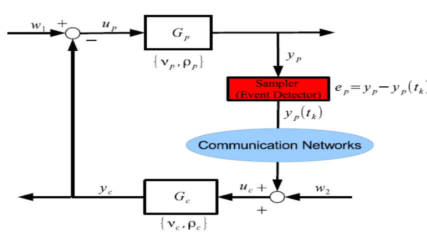
Conclusions

- We considered passivity analysis and passivation problems for feedback interconnected IF-OFP systems.
- Passivity levels were characterized for feedback systems.
- Passivation conditions were provided to obtain required passivity levels in design of nonlinear systems.
- Results can be extended to event-triggered feedback systems.

[Zhu, Xia and Antsaklis, 2014 IFAC]



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Generalized Passivation of Systems with Application to Systems with Input/Output Delays

[Xia, Antsaklis and Gupta, 2014 CDC]
[Xia, Antsaklis and Gupta, ISIS-2014-002]

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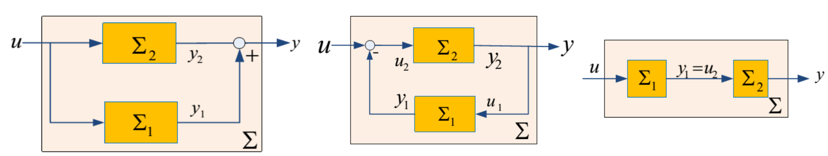
Passivation

- Passivation methods refer to methods that can render a non-passive system passive, e.g. parallel, feedback and series interconnections



(1) Parallel (2) Feedback (3) Series

- Feedback passivation cannot passivate systems that are <u>non-minimum phase</u> or have <u>relative degree larger than one</u>, e.g.

$$\boxed{\frac{s-1}{s+1}} \qquad \boxed{\frac{1}{s^2+s+1}}$$

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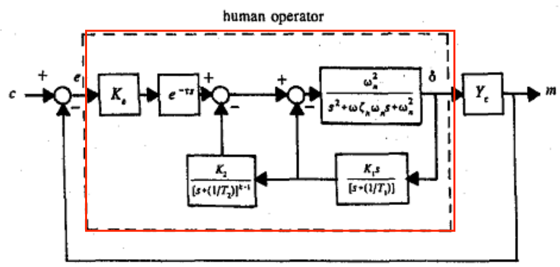
Motivation

- Consider linear systems with input-output delay (e.g. chemical systems, human operators etc.)

$$G(s) = G_0(s)e^{-\tau s}$$

  - G_0 is a SISO, stable, proper, rational transfer function;
  - $\tau > 0$ denotes the transport delay;
  - an example for linear human operator model
  - such systems cannot be passivated through feedback alone.



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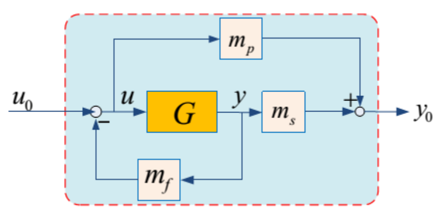
Problem Setup

- The following passivation method can be viewed as a combination of parallel, feedback and series interconnections



$$\begin{bmatrix} u_0 \\ y_0 \end{bmatrix} = M \begin{bmatrix} u \\ y \end{bmatrix} \quad \text{and} \quad M \triangleq \begin{bmatrix} 1 & m_f \\ m_p & m_s \end{bmatrix}$$

- How to select the passivation parameters so that system is passive? In addition, can they improve system $\Sigma_0 : u_0 \to y_0$ performance?

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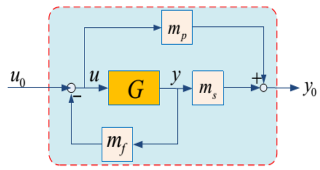
Passivation Results

- **Theorem**: Let system G be finite-gain stable with gain γ. Consider the passivation method as shown in the following figure.



If the passivation are chosen such that

$$1 > m_f \gamma > 0, \quad m_f m_p > m_s > 0,$$

then the passivated system $\Sigma_0 :\ u_0 \to y_0$ is ISP with IFP level

$$\upsilon_0 = \frac{1}{2}\left(m_p + \frac{m_s}{m_f}\right) > 0.$$

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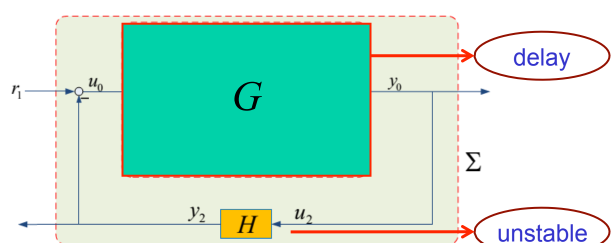
Passivation Results contd

- In the passivity theorem, to guarantee passivity or stability of the closed-loop system, both plant and controller need to be passive, or at least one of them has to be input/output strictly passive.
- If such conditions do not hold (e.g. both systems are non-passive), then the passivation method can be used to guarantee passivity.



$$\upsilon_0 = \frac{1}{2}\left(m_p + \frac{m_s}{m_f}\right) > -\boxed{\rho}$$ OFP of H

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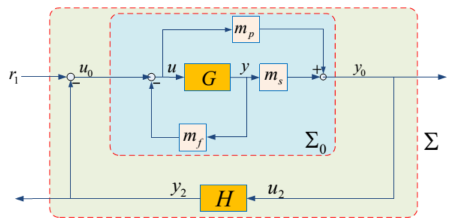
Feedback Interconnections

- **Theorem**: Consider the feedback configuration in the following figure, where r_1 can be seen as the reference to the controller G. Assume that the plant H has an OFP level $\rho < 0$. If the passivation parameters (m_p, m_s, m_f) are chosen such that

$$\upsilon_0 = \frac{1}{2}\left(m_p + \frac{m_s}{m_f}\right) > -\rho, \qquad 1 > m_f \gamma > 0, \; m_f m_p > m_s > 0,$$

then the closed-loop system is output strictly passive. Furthermore, the system is finite-gain stable with gain no larger than $\frac{1}{\rho + \upsilon_0}$.

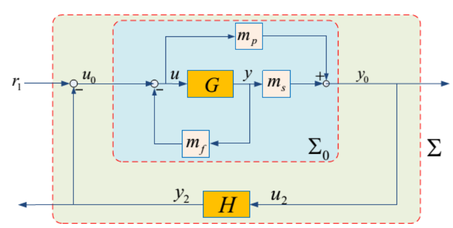


Performance Optimization

- The passivation parameters can be selected to optimize system performance, such as minimizing the tracking error.

tracking error

Minimize:	$J = \int_0^T \|r_1 - y_2\|^2 \mathrm{d}t$
w.r.t:	$m_p, \; m_s, \; m_f$
subject to:	passivation conditions



- If the dynamics for the plant H are known, one can use gradient-based optimization methods.
- If the dynamics for the plant is unknown, unreliable or expensive, one can use non-derivative optimization methods.

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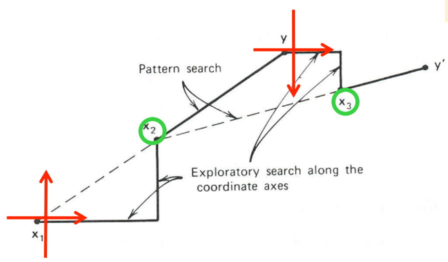
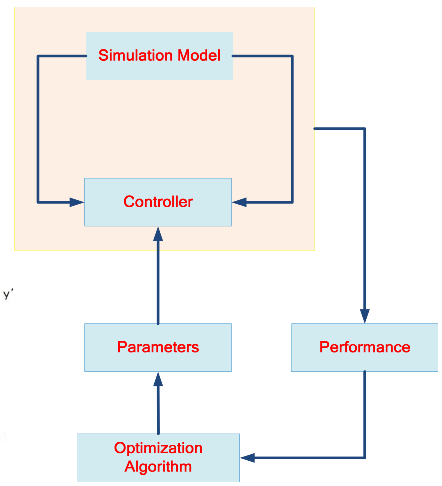
Performance Optimization

- A simulation-based control design (co-simulation) setup
- Optimization algorithm: the method of Hooke and Jeeves





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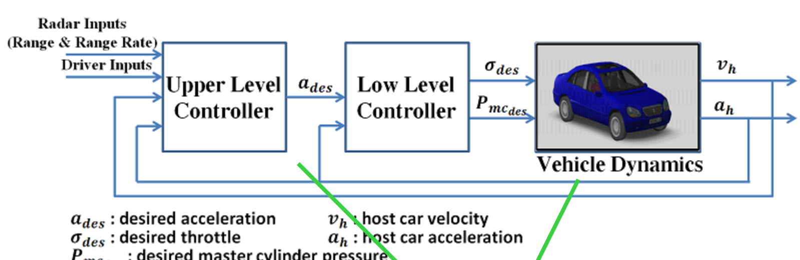
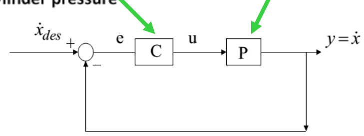
Adaptive Cruise Control Design

- Adaptive cruise control design for automotive systems
  - two control modes: speed control and spacing control
  - a typical value for the time delay: 0.5 seconds





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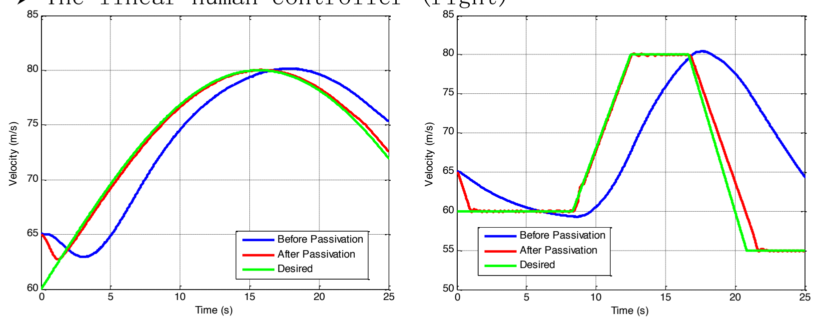
Adaptive Cruise Control Design contd

- Two controllers were considered for minimizing the tracking error
  - PI controller with delay (left)
  - The linear human controller (right)



- The passivation parameters can greatly improve system performance in addition to guaranteeing passivity.

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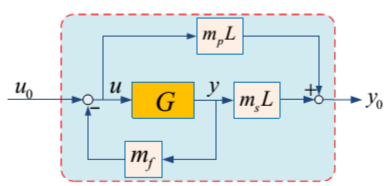
Passivation Using Transfer Functions

- Motivation: braking frequently (left)
- Solution: use low pass filter for passivation (right) $L = \dfrac{0.08s+1}{s+1}$



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Summary of Contributions

- The passivation method represents a combination of series, parallel and feedback interconnections
- Such a general passivation method works for any finite gain stable (linear or nonlinear) systems, e.g. system with input-output delays
- The passivation parameters can guarantee not only passivity but also desired performance
- Validated through simulations in CarSim and Simulink
- On-going and future work may include:
  - Use of non-positive passivation parameters and transfer functions for passivation
  - Extensions to passivation of switched systems
  - Analytic relation between performance and passivity indices

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Passivity/Dissipativity for Hybrid/Switched and DES Systems

- Switched Systems. Definitions and compositionality results. [McCourt, Dissertation, Apr. 2013]
- Hybrid Systems. Definitions.
- DES abstractions of continuous systems. Define granularity to preserve passivity. [Sajja, Gupta and Antsaklis, ISIS-2014-005]
- Approximations. An alternative method to define passive DES and Hybrid systems.

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Passivity and QSR-dissipativity Analysis of a System and its Approximations

[Xia, Antsaklis,Gupta and Zhu, 2014 TAC, submitted]
[Xia, Antsaklis and Gupta, 2013 ACC]
[Xia, Antsaklis and Gupta, ISIS-2012-007]

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Problem Statement

- Problem Setup (input-output mapping):

$$u \longrightarrow \boxed{\Sigma_2} \longrightarrow y_2 = y_1 + \Delta y$$

$$u \longrightarrow \boxed{\Sigma_1} \longrightarrow y_1$$

- Error constraint (for the "worst" case):

$$\boxed{\langle \Delta y, \Delta y \rangle_T \le \gamma^2 \langle u, u \rangle_T + \varepsilon,} \quad \forall u \in U,\ T \ge 0.$$

- For two linear systems given by G_1 and G_2, γ is an upper bound on the H-infinity norm of the difference between two transfer functions $\Delta G = G_1 - G_2$, i.e. if $\|\Delta G\|_{H_\infty} \le \gamma$, then the error constraint holds.

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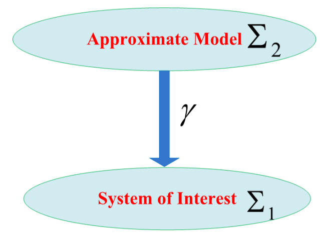
General Result

- Assume that the error γ is 'small'.

$$u \rightarrow \boxed{\Sigma_2} \rightarrow y_2 = y_1 + \Delta y$$

$$u \rightarrow \boxed{\Sigma_1} \rightarrow y_1$$

$$\langle \Delta y, \Delta y \rangle_T \le \gamma^2 \langle u, u \rangle_T + \varepsilon$$



- when an approximate model Σ_2 has an excess of passivity:
  $\rho > 0$ or $\upsilon > 0$
- when an approximate model Σ_2 is not necessarily passive:
  $\rho < 0$ and $\upsilon < 0$

Then certain passivity levels for system Σ_1 can be guaranteed.

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Very Strictly Passive Systems

- Recall the definition for VSP systems:

$$\dot{V} \le u^T y - \upsilon u^T u - \rho y^T y, \quad (\rho > 0, \upsilon > 0)$$

- **Theorem:** Consider the two systems as shown in the following figure. Suppose that the error constraint is satisfied, i.e.

$$\langle \Delta y, \Delta y \rangle_T \le \gamma^2 \langle u, u \rangle_T + \varepsilon,$$

and system Σ_2 is VSP for (ρ, υ). Assume that $\gamma < \rho$ and $\gamma < \upsilon$. If the following condition is satisfied,

$$\gamma^2 - (\rho - \frac{2}{\rho})\gamma + \upsilon^2 - 2 \ge 0,$$

then system Σ_1 is VSP for $(\rho - \gamma, \upsilon - \gamma)$.

$$u \rightarrow \boxed{\Sigma_2} \rightarrow y_2 = y_1 + \Delta y$$

$$u \rightarrow \boxed{\Sigma_1} \rightarrow y_1$$

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(Q,S,R)-dissipative Systems

- Consider a special case for non-passive systems:

$$\dot{V} \le u^T y - \upsilon u^T u - \rho y^T y, \quad \boxed{(\rho < 0, \upsilon < 0)}$$

- **Theorem:** Consider the two systems as shown in the following figure. Suppose that the error constraint is satisfied, i.e.

$$\langle \Delta y, \Delta y \rangle_T \le \gamma^2 \langle u, u \rangle_T + \varepsilon,$$

and system Σ_2 is has passivity levels $(\rho_2 < 0, \upsilon_2 < 0)$. Then Σ_1 has passivity levels (ρ_1, υ_1), if we can find $\xi > 0$ such that

$$\rho_2 - \rho_1 - \xi \rho_1^{\ 2} \ge 0,$$

$$\frac{\gamma^2}{\xi} + \gamma \le \upsilon_2 - \upsilon_1.$$

$$u \rightarrow \boxed{\Sigma_2} \rightarrow y_2 = y_1 + \Delta y$$

$$u \rightarrow \boxed{\Sigma_1} \rightarrow y_1$$

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Particular Approximation Methods

- Linearization around an equilibrium point:
  - γ is determined by the radius of the ball around (0,0).
- Model reduction of linear systems:
  - γ is determined by the Hankel singular values.
- Sampled-data systems:
  - γ is determined by the sampling period.
- Quantization (e.g. logarithmic quantizers):
  - γ is determined by the quantizer parameters.

$$\text{Approximate Model } \Sigma_2 \xrightarrow{\ \gamma\ } \text{System of Interest } \Sigma_1$$

$$\langle \Delta y, \Delta y \rangle_T \le \gamma^2 \langle u, u \rangle_T + \varepsilon$$

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Model Reduction

- Algorithm: truncated balanced realization (TBR)
- Error bound is given by Hankel singular values:

$$\|G_1 - G_2\|_{H_\infty} \le \boxed{2\sum_{i=r+1}^{n} \sigma_i} \longrightarrow \langle \Delta y, \Delta y \rangle_T \le \gamma^2 \langle u, u \rangle_T + \varepsilon$$

- **Corollary:** Let G_1 be a stable LTI system with order n. Let G_2 be a reduced order model of G_1 with order r, obtained using the TBR procedure. Define

$$\gamma = 2\sum_{i=r+1}^{n} \sigma_i, \quad (\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_n),$$

where σ_i is the i th Hankel singular values of system G_1. Assume system G_1 is VSP for (ρ, υ). If $\gamma < \rho$ and $\gamma < \upsilon$, and the condition

$$\gamma^2 - (\rho - \frac{2}{\rho})\gamma + \upsilon^2 - 2 \ge 0$$

is satisfied, then system G_2 is VSP for $(\rho - \gamma, \upsilon - \gamma)$.

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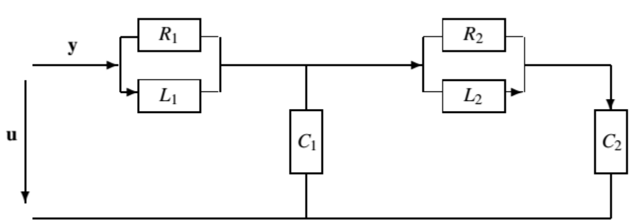
Numerical Example

- Example: an RLC circuit



- The model is given by a 4-th order transfer function

$$G_1 = \frac{s^4 + 1.833s^3 + 1.25s^2 + 0.25s}{s^4 + 3.5s^3 + 2.417s^2 + 0.8333s + 0.1667}$$

$$(R_1 = 1, R_2 = 2, C_1 = 0.5, C_2 = 1, L_1 = 3, L_2 = 4)$$

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Numerical Example contd

- Model reduction using TBR algorithms, we obtain a 2nd order approximate model:

$$G_2 = \frac{s^2 + 1.399s - 0.08816}{s^2 + 3.069s + 0.2669}$$

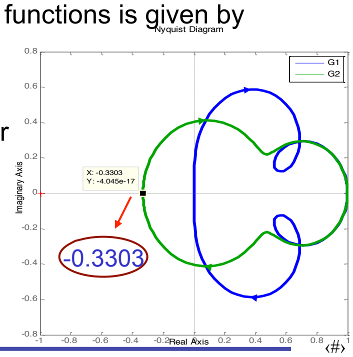
- The difference between the two transfer functions is given by

$$\gamma = 0.3303$$

- According to our results, the IFP level for system G_2 is less than

$$\upsilon_2 < \upsilon_1 - \gamma = -\gamma.$$

- Verified through Nyquist plots:



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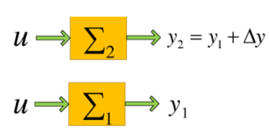
Summary of Contributions

- Passivity properties of a system can be obtained by analyzing its approximations.
- Robustness properties of passivity and dissipativity with respect to modeling uncertainties, etc
- Particular approximation methods ranging from linearization, model reduction, sampling and quantization



- Future Work may include:
  - Control design using approximate models
  - Extension to hybrid dynamical systems, discrete-event systems

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Symmetry and Dissipativity

[Wu, Ghanbari and Antsaklis, 2014 TAC, submitted]
[Wu and Antsaklis, 2011 MED]
[Wu and Antsaklis, 2010 MED]

Symmetry in Systems

- Symmetry: A basic feature of shapes and graphs, indicating some degree of repetition or regularity
  - Symmetry in characterizations of information structure
  - Identical dynamics of subsystems
  - Invariance under group transformation e.g. rotational symmetry
- Why Symmetry?
  - Decompose into lower dimensional systems with better understanding of system properties such as stability and controllability
  - Construct symmetric large-scale systems without reducing performance if certain properties of low dimensional systems hold

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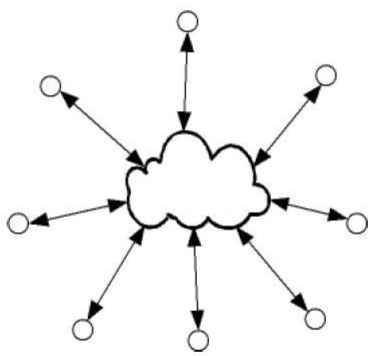
Simple Examples

Star-shaped Symmetry



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

$$u_0 = u_{e0} - Hy_0 - by_1 - \cdots - by_m$$
$$u_1 = u_{e1} - cy_0 - hy_1$$
$$\vdots$$
$$u_m = u_{em} - cy_0 - hy_m$$

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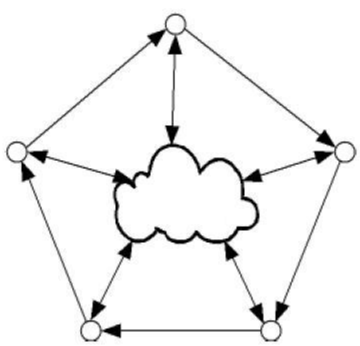
Simple Examples

Cyclic Symmetry



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

$$u_0 = u_{e0} - Hy_0 - by_1 - \mathrm{L} - by_m$$
$$u_1 = u_{e1} - cy_0 - v_1y_1 - v_2y_2 - \mathrm{L} - v_my_m$$
$$\mathrm{M}$$
$$u_m = u_{em} - cy_0 - v_2y_1 - v_3y_2 - \mathrm{L} - v_1y_m$$

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Main Results (1)

Theorem (Star-shaped Symmetry)

Consider a (Q, S, R) – dissipative system extended by *m* star-shaped symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\bar{\sigma}(c^T rc + \beta(\hat{q} - b^T Rb)^{-1}\beta^T)}, \frac{\hat{q}}{b^T Rb}\right)$$

where

$$\hat{Q} = -H^T RH + SH + H^T S^T - Q > 0$$
$$\hat{q} = -h^T rh + sh + h^T s^T - q > 0$$
$$\beta = Sb + c^T s^T - H^T Rb - c^T rh$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

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Main Results (2)

Theorem (Cyclic Symmetry)

Consider a (Q, S, R) – dissipative system extended by *m* cyclic symmetric (q, s, r) – dissipative subsystems. The whole system is finite gain input-out stable if

$$m < \min\left(\frac{\underline{\sigma}(\hat{Q})}{\bar{\sigma}(c^T rc + \beta_m \Lambda^{-1}\beta_m^T)}, \frac{-r\sigma(\tilde{h})\overline{\sigma(\tilde{h})} + s(\sigma(\tilde{h}) + \overline{\sigma(\tilde{h})}) - q}{b^T Rb}\right)$$

where

$$\tilde{h} = circ([v_1, v_2, \ldots, v_m])$$
$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi ij}{m}}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

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Main Results (3)

(cont.)

$$\Lambda = -r\tilde{h}^T\tilde{h} + s(\tilde{h}^T + \tilde{h}) - q \otimes I_m - b^T R b \otimes circ([1,1,\ldots,1])$$

$$\beta = Sb + c^T s^T - H^T Rb - c^T r\tilde{h}$$

$$\beta_m = \underbrace{[\beta\beta\ldots\beta]}_{m}$$

the spectral characterization of $\tilde{h}$ should satisfy

$$\| \sigma(\tilde{h}) - \frac{s}{r} \| < \sqrt{\frac{s^2}{r^2} - \frac{q + mb^T Rb}{r}}$$

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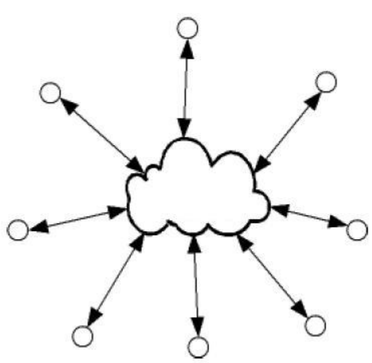
Simple Examples



$$u = u_e - \tilde{H}y$$

$$\tilde{H} = \underbrace{\begin{bmatrix} 0.9 & -0.8 & -0.8 & \cdots & -0.8 \\ -0.8 & 0.1 & 0 & \cdots & 0 \\ -0.8 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -0.8 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m+1}$$

$$Q = q = -I, S = s = 0, R = r = \tfrac{1}{4} I$$

$$\Longrightarrow \quad m < \min(3.11, 6.25) = 3.11$$

Remark: $(-I, 0, \alpha^2 I) - \text{dissipative}$ systems corresponding to systems with gain less or equal to α (here $\alpha = \frac{1}{2}$)

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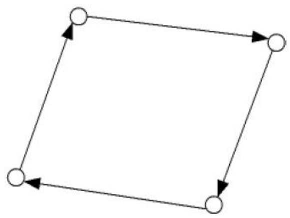
Simple Examples



$$u = u_e - \tilde{H}y \qquad q = 0, s = \tfrac{1}{2}, r = 1$$

$$\tilde{H} = \tilde{h} = \underbrace{\begin{bmatrix} 0.1 & 0.2 & 0 & \cdots & 0 \\ 0 & 0.1 & 0.2 & \cdots & 0 \\ 0 & 0 & 0.1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0.2 & 0 & 0 & \cdots & 0.1 \end{bmatrix}}_{m}$$

The cyclic symmetric system is stable if

$$|\sigma(\tilde{h}) - \frac{s}{r}| = \| \sum_{j=0}^{m-1} v_j e^{\frac{2\pi i j}{m}} \| \le 0.3 < 0.5 = \sqrt{\frac{s^2}{r^2} - \frac{q}{r}}$$

The above stability condition is always satisfied. Also

$$m < \min(+\infty, +\infty)$$

Thus the system can be extended with infinite numbers of subsystems without losing stability.

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Main Results (4)

Theorem (Star-shaped Symmetry for **Passive Systems**)

Consider a passive system extended by *m* star-shaped symmetric passive subsystems. The whole system is finite gain input-output stable if

$$m < \frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(\beta \hat{Q}^{-1} \beta^T)}$$

where

$$\hat{Q} = \frac{H + H^T}{2} > 0$$

$$\hat{q} = \frac{h + h^T}{2} > 0$$

$$\beta = \frac{b + c^T}{2}$$

$$\tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & h & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c & 0 & \cdots & h \end{bmatrix}$$

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Main Results (5)

Theorem (Cyclic Symmetry for **Passive Systems**)

Consider a passive system extended by *m* cyclic symmetric passive subsystems. The whole system is finite gain input-output stable if

$$m < \frac{\underline{\sigma}(\hat{Q})}{\overline{\sigma}(\beta_m \Lambda^{-1} \beta_m^{\ T})}$$

where

$$\hat{Q} = \frac{H + H^T}{2} > 0 \qquad \Lambda = \frac{\tilde{h} + \tilde{h}^T}{2} \qquad \tilde{H} = \begin{bmatrix} H & b & \cdots & b \\ c & & & \\ \vdots & & \tilde{h} & \\ c & & & \end{bmatrix}$$

$$\beta = \frac{b + c^T}{2} \qquad \beta_m = \underbrace{[\beta\beta\ldots\beta]}_{m}$$

$$\sigma(\tilde{h}) = \sum_{j=1}^{m} v_j \lambda_i^j = \sum_{j=1}^{m} v_j e^{\frac{2\pi i j}{m}} \qquad \tilde{h} = circ([v_1, v_2, \ldots, v_m])$$

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Symmetry and Passivity/Dissipativity

- **Approximate symmetries. Robustness of results.**
- **The subsystem dynamics may not be identical as long as they satisfy the same q, s, r inequalities.**
- **Results still valid when the strengths of the interconnections (b, c) are not exactly the same.**

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Additional Passivity/Dissipativity Results

- Event-triggered control for networked systems using passivity. Event triggered control is used to reduce communication in networked control systems. Output Synchronization [Yu and Antsaklis 2011 CDC]
- Passivity of Systems in Series. A Passivity Measure Of Systems In Cascade Based On Passivity Indices [Yu and Antsaklis 2010 CDC]
- Compensating for Quantization [Zhu, Yu, McCourt, 2012 HSCC]
- Linearization. Preserving passivity. [Xia, Antsaklis, Gupta and McCourt, 2014 TAC, to appear]
- Model Predictive Control [Yu, Zhu, Xia, 2013 ECC]
- Shravan Sajja, Vijay Gupta and Panos. J. Antsaklis, Passivity based Supervisory Control, [ISIS Technical Report ISIS-2014-005, July 2014. Revised March 2015].

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Concluding Remarks

- Main points
  - **New ways of thinking needed to deal effectively with the CPS problems. New ways to determine research directions.**
  - Passivity/Dissipativity and Symmetry are promising
  - Need deeper understanding of fundamentals that cut across disciplines.

    CPS, Distributed, Embedded, Networked Systems. Analog-digital, large scale, life cycles, safety critical, end to end high-confidence.
  - **Need to expand our horizons. Control Systems at the center.**
  - Collaborations with, build bridges to Computer Science, Networks, Biology, Physics. Also Sociology, Psychology…

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